For many years, the importance of problem solving in mathematics education has been well recognized. As Thompson and colleagues (2009) noted, “Good problems challenge students to develop and apply strategies, serve as a means to introduce new concepts, and offer a context for using skills. Problem solving is not a specific topic to be taught but permeates all mathematics” (p. 2). In recent years, there has been a greater emphasis in research and curriculum documents on the important role played in problem solving by cognitively demanding tasks (Stein et al. 2009).

An example of a cognitively demanding task for grade 6 students is: How much money would we need to cover one square meter with dimes? It is possible to respond to the task trivially, but some substantial mathematics is also involved in investigating options other than the most obvious configuration, such as whether the amount would be maximized by having the coins arranged into a parallelogram. There is substantially more potential for mathematics learning by going beyond the obvious and persisting at the challenge of the task.

It is important for students to learn mathematics, but currently too many miss out on the opportunities that successful learning creates (Australian Curriculum Assessment and Reporting Authority [ACARA] 2009; U.S. Department of Education 2008). While it is possible for everyone to learn mathematics, it takes concentration and effort over an extended period of time for students to build the connections between topics, to understand the coherence of mathematical ideas, and to be able to transfer learning to practical contexts and new topics. We use the term persistence to describe student actions that include students’ concentrating, applying themselves, believing that they can succeed, and making an effort to learn, and we term the tasks that are likely to foster such actions cognitively demanding, or challenging in that they allow the possibility of sustained thinking, decision making, and some risk taking by the students.
The notion of persistence is encapsulated in widely used principles of effective teaching that recommend that teachers communicate high expectations to students. This involves teachers posing challenging tasks and encouraging students to task risks in their learning, to justify their thinking, to make decisions, and to work with other students (Stein et al. 2009; Sullivan 2011). Indeed, the first standard in the eight Standards for Mathematical Practice in the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices [NGA Center] and Council of Chief State School Officers [CCSSO] 2010) focuses on making sense of problems and persevering to solve them.

In encouraging persistence, it is also critical for teachers to understand the motivational characteristics of their students, particularly those characteristics that contribute to increased persistence, valuation of mathematics as a useful domain, and an orientation toward taking on challenging tasks (Middleton and Jansen 2011).

Yet two concurrent projects with which we have been involved in recent years found that, on one hand, teachers seemed reluctant to pose challenging tasks to students, and on the other hand, students seemed to resist engaging with those tasks and exerted both passive and active pressure on teachers to overexplain tasks or to pose simpler ones (Sullivan, Clarke, and Clarke 2013).

Challenging tasks are important for all students. Pogrow (1988) warned that by protecting the self-image of underachieving students through giving them only “simple, dull material” (p. 84), teachers actually prevent them from developing self-confidence. He maintained that it is only through success on complex tasks that are valued by the students and teachers that such students can achieve confidence in their abilities. There will be an inevitable period of struggling while the students begin to grapple with problems, but Pogrow asserted that this “controlled floundering” is essential for students to begin to think at higher levels.

### Our Project

The Encouraging Persistence Maintaining Challenge Project (EPMC) is researching a range of issues, including the kinds of teacher practices that might encourage students to persist when working on challenging tasks in mathematics. EPMC is a collaborative project involving university researchers and classroom teachers. It is funded through an Australian Research Council Discovery Project (DP110101027) and is a collaboration between the authors and their universities.

The study can be considered design research, described by Cobb and colleagues (2003) as “engineering particular forms of learning and systematically studying those forms of learning within the context defined by the means of supporting them” (p. 9).

A framework that guided our thinking about using challenging tasks to build student persistence was proposed by Stein and colleagues (2009) as shown in figure 7.1. It is the actions of the teacher in setting up the task that are of particular interest to us in this article.

![Fig. 7.1. The mathematical tasks framework (Stein et al. 2009)](image_url)
Creating a Classroom Culture That Encourages Students to Persist   ■ CHAPTER 7

A Classroom Example of Strategies to Encourage Persistence

To elaborate helpful teacher actions that are emerging as important in our project, we now present a classroom scenario using a lesson with a series of closely related tasks, which we call Division with Remainders. This scenario is in fact an amalgam of several lessons in which these tasks were used in different contexts. We outline the ways the mathematical tasks were set up by the teacher. As we describe the various stages of the lesson, we also include reflections on teacher decision making and emerging insights.

Setting the Scene

The teacher told students that today they would be working on a series of quite challenging tasks (see appendix A), but he was confident that if the students persisted, they would find the tasks quite interesting, and the struggle would lead to worthwhile learning.

The teacher then asked for a student to suggest a situation where a remainder would be required. One girl offered “7 apples, shared between 3 people,” and explained that this would be “2 apples each with one remainder.” The teacher wrote $7 ÷ 3 = 2 r 1$ as guided by the student, but noted that the remainder of one had little meaning, as you would not give a person “remainder one of an apple.”

He explained that the focus of the day’s lesson was on the idea that the context of a division problem determines the way in which the remainder is expressed. Further, he said that the lesson would involve students working in pairs on a series of division tasks, all of which involved remainders. Calculators would be freely available, but the students would need to consider how best to use them and how to make sense of the result.

We have found that a teacher’s enthusiasm for the task and a belief in students’ capacity to succeed are important. Middleton and Jansen (2011) noted that motivation is “in the moment” and that students are motivated through engagement in tasks that pique their imagination, challenge their abilities, and afford them the opportunity to learn at an optimum level. Middleton and Jansen claimed that “adjusting norms and practices can have a dramatic effect on students’ interests” (p. 41). They note, for example, that “through careful instructional design, appropriate feedback and attention to students, or the management of a discussion to incorporate input from a variety of learners” (p. 43), the learning experiences of students can change from negative or neutral to positive.

We also believe that it is important not only to talk with students about challenging tasks, including the reasons they are used, and acknowledge that they require persistence but also to indicate that persistence brings rewards. We explain that struggle is important for students if real learning is to take place. As Hiebert and Grouws (2007) noted, “We use the word struggle to mean that students expend effort to make sense of mathematics, to figure something out that is not immediately apparent. We do not use struggle to mean needless frustration or extreme levels of challenge created by nonsensical or overly difficult problems” (p. 387).

Planning in Pairs

Before the students started work, the teacher encouraged them to identify someone with whom they would be likely to work well and to talk on the floor with that student about the first few tasks on the sheet: “I don’t want you to start working on the tasks, but just to talk about how you
will approach the tasks when you return to your tables.” The teacher explained that he would be pulling the class back together after each pair had some time to work on two or three of the tasks, to briefly share progress. He emphasized that as well as finding solutions, the students needed to explain their reasoning in writing: “You really know if you understand something in mathematics when you can explain it to someone else.”

To maximize the time that students have to work on the tasks, we have found that a brief introduction prior to student work is desirable, including an overview of how the lesson will play out, key stages and expectations, and timing of these.

**Getting Down to Work**

Having had the chance to discuss how they might approach the tasks, many students noticed an important feature of the tasks, namely that all tasks involved division and the same numbers. Their initial thoughts were that having the same numbers and access to calculators must make things easy, but they were soon to find that this was not necessarily the case. Each task brought its own challenge, as they had to think about expressing the remainder in a way that responded to the context.

During this working time, the teacher spent most of the initial ten minutes or so just watching and listening and resisting the temptation to “tell.” He noted the difficulty some students were having in interpreting the 0.75 part of the 35.75 resulting from using a calculator to divide 1144 by 32. However, he also knew that it was important for them to struggle in order to make sense of the mathematics, to have ownership of the thinking, and therefore to eventually have a sense of achievement.

**A Brief Coming Back Together**

The teacher then drew the class back to the floor. First, he wrote 35.75 on the board and asked what the 0.75 meant. He wrote down the many answers that students offered, including $\frac{3}{4}$, $\frac{6}{8}$, 75%, 75/100, 0.7 + 0.05. Without dwelling on this, the teacher then asked for some of the answers that pairs had found for the buses task (see appendix A, problem 1a). Some of these are shown in figure 7.2.

![Image](image_url)

*Fig. 7.2. Some answers students offered to question 1a (the buses task)*
Creating a Classroom Culture That Encourages Students to Persist

It was clear from the range of answers that many students were approaching the context in a meaningful way, while others were ignoring it. Some used the calculator to divide 1144 by 32, others used estimating and checking, while some added 32 repeatedly by hand or calculator until they were close to 1144. In a short period of time, there was consensus about the correct answer to the first question. The teacher then reiterated the lesson focus—that the context of a division problem determines the way in which the remainder is expressed—and encouraged the students to continue their work.

We have learned that it is important to make this brief recall of the group just that: brief. Students need serious time to work on these tasks (around forty minutes of concentrated work, from our experience), and teachers can often fall into the trap of not providing sufficient time for the students to persist.

Working through the Other Problems

The teacher continued to move around the class, observing and listening, asking questions as necessary, pushing students to justify their thinking, and occasionally urging them to “write that down.” The teacher was making many decisions “on the run” in the way described by Lampert (2001):

I watch and listen, sometimes asking questions about what I see or hear. Even when I am “just watching,” I am also teaching a lesson about the study of mathematics. I deliberately talk and stand and look in ways that intend to communicate that it matters to me what they say and do, even if I do not comment on it. . . . I spent about 20 minutes out of the 30 small-group part of the lesson just watching and listening, and the rest of the time interacting. As I walked around watching students work, the teaching I did was constructed in response to whatever I saw or heard on the spot. (Lampert 2001, pp. 122–23)

Several issues emerge when these kinds of challenging tasks are given. One issue is the range of levels of understanding in the class. How do we provide appropriate support for the students who are struggling to make a start, and how do we also challenge those who move through the tasks quickly? We have found that the brief class discussion partway through the lesson is often helpful for those who are struggling (and its brevity is not a problem for the very capable students). But we also prepare what have been called enabling prompts (Sullivan 2011), which in this case involve the same context (buses), but with smaller numbers (e.g., if we had 70 passengers with 32 on each bus, how many buses would we need?). The intention is not that students who struggle are given a different set of experiences for the whole lesson, but rather a slightly varied initial task to get them on track before they return to the main tasks. In the same way, we prepare extending prompts, which are challenging for the very capable students. In this lesson, questions 2, 3, and 4 provided that challenge, where students are encouraged to generalize and pose their own problems. In surveys of teachers who have used many challenging tasks, when asked to list key strategies for encouraging persistence, they frequently mention the power of enabling and extending prompts (Clarke et al. 2014).

Some teachers with whom we work believe that their planning should involve preparation of different sets of tasks for different levels of “ability.” Our strong preference, however, is the use of these enabling and extending prompts in a mixed-ability setting. In this context, teachers frequently express surprise at what their “weaker” students can achieve given challenging tasks and high expectations.
Using Research to Improve Instruction

Another challenge for students is our requirement for written explanations of student reasoning. Students are often reluctant to complete these sections, but the teacher needs to keep emphasizing the importance of being able to justify decisions made and the fact that writing ideas down can help to clarify them and provide a record for later discussion. It takes time for students to feel confident in doing this, but the effort is worthwhile.

Pulling the Lesson Together

The teacher, before and during the lesson, moved through the stages advocated by Stein and colleagues (2008), all designed to make the latter part of the lesson most fruitful:

• anticipating likely student responses to cognitively demanding tasks (prior to the lesson);
• monitoring students’ responses to the tasks during the explore phase;
• selecting particular students to present their mathematical responses during the discussion and summarize phase;
• purposefully sequencing the student responses that will be displayed; and
• helping the class make the mathematical connections between different students’ responses.

(p. 321)

Students’ errors and misconceptions very much came to the fore during the discussion at the end of the lesson, but in a helpful and productive way. For example, misconceptions in relation to 35.75 hours arose, as some students took this to be 35 hours and 75 minutes (therefore 36 hours and 15 minutes) or 3575 minutes!

We believe that a flexible approach is needed and that the final part (discuss and summarize) may take place at multiple points in the lesson, but also might take place the following day. We often find that students need the remainder of the first lesson to work through as many of the tasks as they can and that an extended discussion is sometimes more usefully placed at the start of the next lesson. There are many reasons why this discuss and summarize phase is important, of course: gathering evidence for general findings; summarizing the work of the day; celebrating students’ learning; learning from each other, including learning from errors; and raising possibilities for future mathematical exploration.

What Do Observing Teachers Notice about Persistence and Challenging Tasks?

In our context, we have the chance for many teachers to observe the teaching of others. We ask teachers to record any changes they are considering making in their own teaching as a result of what they have observed. Here is a sample of comments from teachers about what they have learned through teaching challenging tasks and observing others doing so:

• “Don’t immediately jump in to help them—wait time is important.”
• “I’m trusting some struggling students with greater challenge.”
• I say to students, “I’m here to help you by asking you questions and changing your thinking a little bit, and that will help rather than just giving you a lifeline out.”
Conclusion

For worthwhile learning in mathematics, all students need mathematically appropriate, engaging and cognitively demanding tasks. At the same time, the decisions that the teacher makes (in planning and on the run) can make a considerable difference in how the task plays out, the level of persistence shown by students, and the resulting learning, cognitively and affectively.

From our experiences in using the tasks described in this article and in considering insights from teachers and the project team, we offer the following list of strategies for encouraging persistence on challenging tasks:

- Attempt to connect the task with students’ experiences.
- Explain to students how to work, including the type of thinking in which they are expected to engage and what they might later report to the class.
- Communicate enthusiasm about the task, including encouraging students to persist with it.
- Build a classroom climate that encourages risk taking. Expect students to succeed. Errors are part of learning, and students can learn even if they do not complete the task.
- Structure the lesson to ensure that students have adequate time to work on the challenging task.
- Clarify processes and expectations for recording, including encouraging students to make appropriate notes along the way.
- Move around the class, predominantly observing students at work, selecting students who might report and giving them a sense of their role, intervening only when necessary to seek clarification of potential misconceptions, to support students who cannot proceed, and to challenge those who have completed the task.
- Allow time for lesson review so that students see the strategies of other students and any summaries from the teacher as learning opportunities.

Student persistence is important. As one student wrote in a reflection:

“We do learn more when we’re confused and we’ve got to work our way out of it.”
Using Research to Improve Instruction

**Appendix A**

**Division Problems with Remainders**

Name(s) .............................................  Year ........  School .................................

1. Work out the answers to each of these problems. You can use a calculator, but you must explain your reasoning. (Hint: all the answers are different in some way.)

   a. We need to book buses to take all the students in the school to a concert. There are 1144 students, and each bus can take 32 students. How many buses do we need to order?

      Answer: .............................................
      Our reasoning:

   b. We are making up packets of chocolates. Each packet must have exactly 32 chocolates. If we have 1144 chocolates, how many complete packets can we make?

      Answer: .............................................
      Our reasoning:

   c. Our basketball club won a prize of $1144. The 32 members decided to share the prize exactly between them. How much money will each of them get?

      Answer: .............................................
      Our reasoning:

   d. The train from Melbourne to Sydney travels at an average speed of 32 km/hr. How long would it take to travel the 1144 km to Sydney if the train does not stop?

      Answer: .............................................
      Our reasoning:
e. Our year level of 32 students together won a prize of 1144 pizzas. If we share the prize equally, how much pizza do we each get?

Answer: .........................................................
Our reasoning:

f. A dairy farm produced 1144 liters of milk and has 32 containers in which to store the milk. If the containers are filled exactly, how much milk should go into each container?

Answer: .........................................................
Our reasoning:

g. There are 1144 people who need to cross a crocodile-infested river. The ferry can carry 32 people each trip. If everyone is in a hurry to cross the river, how many people will be left for the last trip?

Answer: .........................................................
Our reasoning:

2. In what ways are these problems similar to each other?

3. In what ways are these problems different from each other?

4. Make up another problem to add to this set.
References


